

# Fieldhead Carr Primary School

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Teaching and Learning Governing Body Meeting

January 2016

## Calculations Policy

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### Summary

This policy outlines the calculations methods that are to be adopted consistently across all classes for Fieldhead Carr Primary School.

### Recommendation

This is a new policy that has been written to address the challenges of the new mathematics curriculum. We recommend that the policy be read in full.

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Name	E. Carr and S. Cooke
Job title	Mathematics Leaders
Date	February 2016



**Fieldhead  
Carr**  
PRIMARY SCHOOL

# Maths Calculation Policy

January 2016

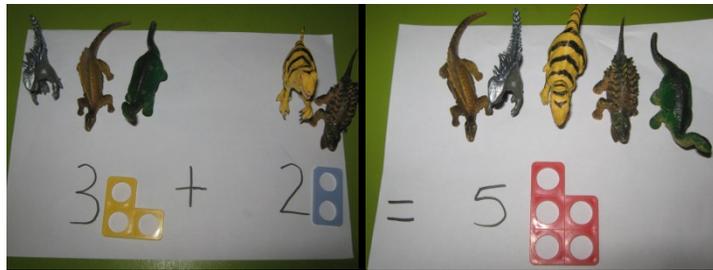
## Progression in Addition

These notes show the stages in building up to a compact, efficient method for addition. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

### Stage 1

#### Practical Addition

Children should add single digit numbers together using practical objects. Children will first add by counting all the objects. Later (when they are able to subitise) they will start with the largest group of objects and count on.



Children should:

1. Use practical objects such as dinosaurs, toy cars, toy sheep etc.
2. Use mathematical representations of numbers e.g. numicon, counters, unifix cubes.

#### Children need to be able to:

- Have one to one correspondence
- Reliably count objects up to 20.
- Recognise numerals up to 20.
- Say one more than any number up to 20.
- Subitise up to 5 objects. (Instantly recognise how many there are without having to count).

#### Key Vocabulary:

Add, more, and, make, altogether, total, equals.

### Stage 2

#### Number lines/Hundred Square

##### 1. Numbered number lines and hundred square

Children should be able to count on using a numbered number line. They should first be able to count on in ones, then tens and then other numbers. Children should also be able to use a hundred square to count on (before using a hundred square to do activities such as cutting one up into rows of ten and placing it out like a number line, then putting it back together as a number square. This will ensure that children understand the link between a number line and a hundred square).

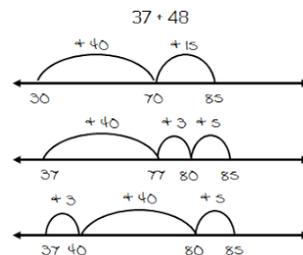
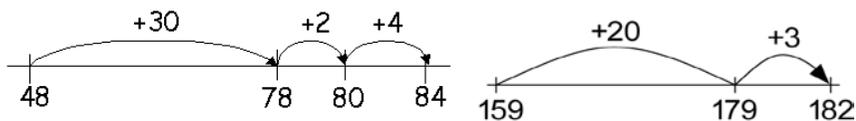
##### 2. Empty number lines

The empty number line helps to record the steps on the way to calculating the total. The steps often bridge through a multiple of ten. Allow children to experiment with the order of adding to allow them to understand that addition can be done in any order. Eventually refine this to starting with the largest

#### Children need to be able to:

- Read and write numbers to 100 in numerals.
- Recall number bonds to 20 and addition facts within 20.
- Show that adding can be done in any order (the commutative law).
- Recognise that addition is the inverse of subtraction.
- Secure place value of two digit numbers.
- Solve one step problems involving addition using practical resources, pictorial representation and number lines.

number, adding the tens and then adding the ones.  
e.g.  $48 + 36 = 84$



### Stage 3

#### Expanded Column Method

The expanded column method leads children on to the compact column method. It should be taught using base ten apparatus and it is important that children can do this practically before they start to record their method. This is an essential step as it allows children to later understand the compact method.

e.g.  $47 + 38 = 85$

	<table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 10px;">T</td> <td>Ones</td> </tr> <tr> <td style="padding-right: 10px;">40</td> <td>7</td> </tr> <tr> <td style="padding-right: 10px;">+ 30</td> <td>8</td> </tr> <tr> <td colspan="2" style="text-align: center;">+</td> </tr> <tr> <td colspan="2" style="border-top: 1px solid black; border-bottom: 1px solid black;"></td> </tr> </table>	T	Ones	40	7	+ 30	8	+				<p>Children should partition both numbers and create these numbers using base ten. They should then set them out on a place diagram mat as shown in the photo.</p>
T	Ones											
40	7											
+ 30	8											
+												
	<table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 10px;">T</td> <td>Ones</td> </tr> <tr> <td style="padding-right: 10px;">40</td> <td>7</td> </tr> <tr> <td style="padding-right: 10px;">+ 30</td> <td>8</td> </tr> <tr> <td colspan="2" style="text-align: center;">+</td> </tr> <tr> <td style="border-top: 1px solid black; border-bottom: 1px solid black;">70</td> <td style="border-top: 1px solid black; border-bottom: 1px solid black;">+ 15</td> </tr> </table>	T	Ones	40	7	+ 30	8	+		70	+ 15	<p>Children should group all the ones together and all the tens together.</p>
T	Ones											
40	7											
+ 30	8											
+												
70	+ 15											

#### Key Vocabulary:

Add, more, and, make, altogether, total, equals, plus, sum, addition, partition, count on, tens boundary.

#### Children need to be able to:

- Have a secure understanding of place value up to 1000.
- Estimate answers.
- Understand subtraction as the inverse of addition.
- Read and write numbers in words and digits up to 1000.
- Solve two step problems (including missing number problems) involving addition.
- Add multiples of ten together.

#### Key Vocabulary:

Add, more, and, make, altogether, total, equals, plus, sum, addition, partition, count on, tens boundary, hundreds boundary, increase, exchange, carry.

#### Progression

1. Add two 2 digit numbers together without exchange e.g.  $34 + 42$
2. Add two 2 digit numbers together

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T	Ones												
40	7												
+ 30	8												
70	+ 15												
1	5												
	<table style="border-collapse: collapse; margin-left: 20px;"> <tr><td style="padding-right: 20px;">T</td><td>Ones</td></tr> <tr><td style="padding-right: 20px;">40</td><td>7</td></tr> <tr><td style="padding-right: 20px;">+ 30</td><td>8</td></tr> <tr style="border-top: 1px solid black;"><td style="padding-right: 20px;">80</td><td>+ 5</td></tr> <tr><td style="padding-right: 20px;">+</td><td></td></tr> </table>	T	Ones	40	7	+ 30	8	80	+ 5	+		<p>Children should then count up how many tens and how many ones they have. They can then regroup them to attain their answer.</p>	
T	Ones												
40	7												
+ 30	8												
80	+ 5												
+													

<p><b>Stage 4</b></p> <p><b>Compact column method</b></p> <p>This is the formal standard method of addition. This method should be taught when children are completely confident in using the expanded column method and can prove this by using base ten apparatus. This expanded method is shortened: when the column total is a two digit number, the tens (or hundreds) are regrouped to the next column. e.g. <math>789 + 642 = 1431</math></p> <table style="margin-left: 40px; border-collapse: collapse;"> <tr><td style="padding-right: 10px;">Th</td><td style="padding-right: 10px;">H</td><td style="padding-right: 10px;">T</td><td>Ones</td></tr> <tr><td></td><td style="text-align: center;">7</td><td style="text-align: center;">8</td><td style="text-align: center;">9</td></tr> <tr style="border-top: 1px solid black;"><td></td><td style="text-align: center;">6</td><td style="text-align: center;">4</td><td style="text-align: center;">2</td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">4</td><td style="text-align: center;">3</td><td style="text-align: center;">1</td></tr> <tr style="border-top: 1px solid black;"><td style="text-align: center;">1</td><td style="text-align: center;">1</td><td></td><td></td></tr> </table> <p><u>Progression</u></p> <ol style="list-style-type: none"> <li>1. Addition of two, three and four digit numbers. (Two or more numbers).</li> <li>2. Addition of decimal numbers to 1,2 and 3 decimal places. (Including amounts of money and other measures).</li> <li>3. Addition of decimal numbers where the two numbers have a different number of decimal places e.g. <math>1.78 + 54.2</math></li> </ol>	Th	H	T	Ones		7	8	9		6	4	2	1	4	3	1	1	1			<p><b>Children needs to be able to:</b></p> <ul style="list-style-type: none"> <li>• Have a secure understanding of the expanded column method.</li> <li>• Have a secure understanding of place value to 10 000 000.</li> <li>• Have a secure understanding of decimal numbers to 3 decimal places.</li> <li>• Understand subtraction as the inverse of addition.</li> <li>• Solve complex multi-step problems (including missing number problems) involving addition.</li> <li>• Estimate answers and use this to check their answer.</li> <li>• Have fluent mental addition skills.</li> </ul> <p><b>Key vocabulary:</b> Add, more, and, make, altogether, total, equals, plus, sum (only for addition), addition, partition, count on, tens boundary, hundreds boundary, increase, exchange, carry, decimal, decimal point, tenths, hundredths, thousandths, inverse</p>
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	7	8	9																		
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1	4	3	1																		
1	1																				

These notes show the stages in building up to a compact, efficient written method for subtraction. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

### Progression in Subtraction

These notes show the stages in building up to a compact, efficient written method for subtraction. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

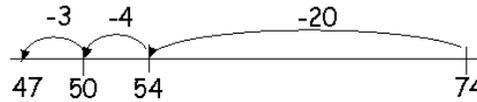
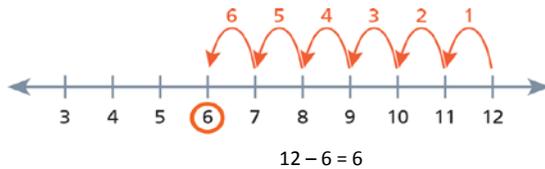
<p><b>Stage 1</b></p> <p><b>Practical Subtraction</b></p> <p>Children should be able to take away objects from a group of objects and be able to say how many they started with, how many they took away and how many they have left.</p> <p>e.g. The child starts with 3 dinosaurs, removes one and knows that there are two left.</p>  <p>Children should:</p> <ol style="list-style-type: none"> <li>1. Use practical objects such as dinosaurs, toy cars, toy sheep etc.</li> <li>2. Use mathematical representation of number e.g. numicon (cover up what is being taken away using subtraction covers).</li> </ol>	<p><b>Children need to be able to:</b></p> <ul style="list-style-type: none"> <li>• Have one to one correspondence.</li> <li>• Reliably count objects to 20.</li> <li>• Recognise numerals up to 20.</li> <li>• Count backwards from 10.</li> <li>• Say one less than any number up to 20.</li> <li>• Subitise up to 5 objects (instantly recognise how many there are without counting).</li> </ul> <p><b>Key Vocabulary:</b> Take, take away, less, fewer, how many left, equals.</p>
<p><b>Stage 2</b></p> <p><b>Number lines/Hundred squares</b></p> <ol style="list-style-type: none"> <li>1. <a href="#">Numbered number lines and hundred squares</a> Children should be able to count back using a numbered number line. This should first be in ones, then tens and then other</li> </ol>	<p><b>Children need to be able to:</b></p> <ul style="list-style-type: none"> <li>• Read and write numbers to 100 in numerals.</li> <li>• Recall number bonds to 20 and subtraction facts within 20.</li> <li>• Understand that subtraction</li> </ul>

numbers. Children should also be able to use a hundred square to count back (before using a hundred square do activities such as cutting one up into rows of ten and placing it out like a number line, then putting it back together as a number square. This will ensure that children understand the link between a number line and a hundred square).

## 2. Empty number lines

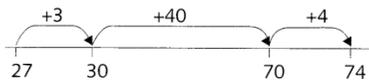
These are a useful way of modelling processes such as bridging through multiples of ten. Children should first be taught to count back on an empty number line.

e.g.  $74 - 27 = 47$



Once children are familiar with subtraction as being the inverse of addition (see separate section about the inverse). Children should be taught to use the empty number line to count on.

e.g.  $74 - 27 = 47$



These steps may be recorded in a different order or combined. With practice, children will record less information and will be able to decide when it is appropriate to count back and count on.

- can not be done in any order.
- Recognise that subtraction is the inverse of addition.
- Have secure place value of two digit numbers.
- Partition and recombine numbers.
- Solve one-step problems involving subtraction.

### Key Vocabulary:

Take, take away, less, fewer, how many left, equals, partition, difference, count back, count on, less than, subtract, minus, tens, ones.

## Stage 3

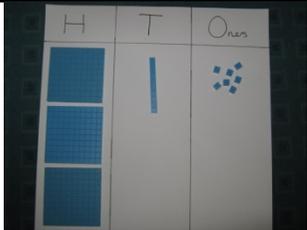
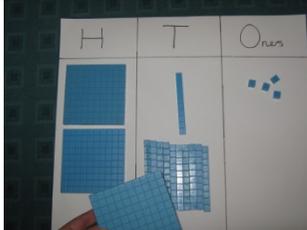
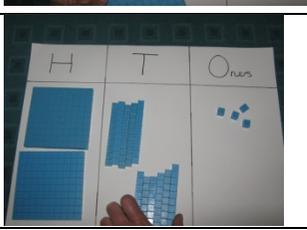
### Expanded column method

The expanded column method is a stepping stone towards the compact column method. It should be taught practically, using base ten apparatus, before children start to record it. Include numbers where zero is a place holder (e.g.  $504 - 173$ ).

e.g.  $317 - 263 = 54$

### Children need to be able to:

- Have a secure understanding of place value up to 1000.
- Estimate answers.
- Understand subtraction as the inverse of multiplication.
- Read and write numbers in words and numerals up to

	<table border="0"> <thead> <tr> <th>H</th> <th>T</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>300</td> <td>10</td> <td>7</td> </tr> <tr> <td>- 200</td> <td>60</td> <td>3</td> </tr> <tr> <td colspan="3"><hr/></td> </tr> </tbody> </table>	H	T	Ones	300	10	7	- 200	60	3	<hr/>			Children create the number that they will be taking away from, using base ten and place it on a place value mat.	<ul style="list-style-type: none"> <li>1000.</li> <li>Solve two-step problems involving subtraction.</li> <li>Subtract multiples of ten from one another.</li> </ul> <p><b>Key Vocabulary:</b> Take, take away, less, fewer, how many left, equals, partition, difference, count back, count on, less than, subtract, minus, exchange, decrease, tens, ones.</p>		
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	<table border="0"> <thead> <tr> <th>H</th> <th>T</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td><sup>200</sup>300</td> <td>110</td> <td>7</td> </tr> <tr> <td>- 200</td> <td>60</td> <td>3</td> </tr> <tr> <td colspan="3"><hr/></td> </tr> <tr> <td></td> <td></td> <td>4</td> </tr> </tbody> </table>	H	T	Ones	<sup>200</sup> 300	110	7	- 200	60	3	<hr/>					4	Starting with the ones, children take away 3 from the 7 ones. Moving onto the tens, children realize that they can not take six tens from one ten so they exchange a hundred for ten tens. It is important that children do this physically.
H	T	Ones															
<sup>200</sup> 300	110	7															
- 200	60	3															
<hr/>																	
		4															
	<table border="0"> <thead> <tr> <th>H</th> <th>T</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td><sup>200</sup>300</td> <td>110</td> <td>7</td> </tr> <tr> <td>- 200</td> <td>60</td> <td>3</td> </tr> <tr> <td colspan="3"><hr/></td> </tr> <tr> <td>0</td> <td>50</td> <td>4</td> </tr> </tbody> </table>	H	T	Ones	<sup>200</sup> 300	110	7	- 200	60	3	<hr/>			0	50	4	Children can now remove 6 tens from 11 tens and then remove the two hundreds. Children will be left with their answer.
H	T	Ones															
<sup>200</sup> 300	110	7															
- 200	60	3															
<hr/>																	
0	50	4															

#### Stage 4

#### Compact column method

This is the formal standard method of subtraction. This should only be taught when children are completely confident in using the expanded column method and can prove this using base ten apparatus.

e.g.

$$74 - 27 = 47$$

$$\begin{array}{r} \overset{6}{7}4 \\ - 27 \\ \hline 47 \end{array}$$

$$563 - 271 = 292$$

$$\begin{array}{r} \overset{4}{5}163 \end{array}$$

#### Children need to be able to:

- Have a secure understanding of the expanded column method.
- Have a secure understanding of place value to 10 000 000.
- Have a secure understanding of decimal numbers to 3 decimal places.
- Understand addition as the inverse of subtraction.
- Solve complex multi-step problems involving subtraction (including missing number problems).
- Estimate answers and use this to check answer.
- Have fluent mental subtraction skills.

$$\begin{array}{r} - 271 \\ \hline 292 \end{array}$$

It is important that children say "16 tens take 7 tens" not "16 -7". Use the term "regroup" not borrow.

Progression

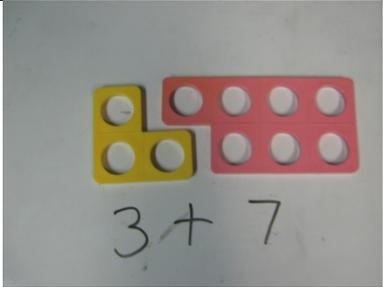
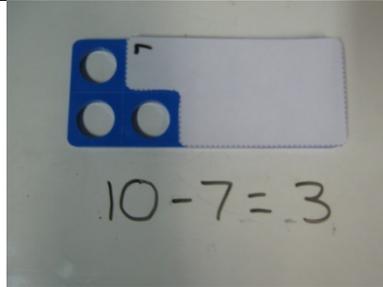
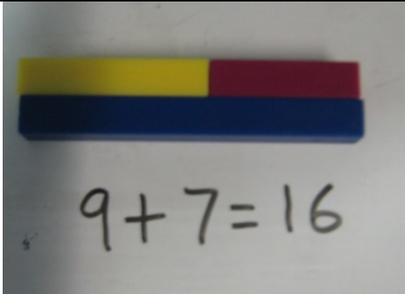
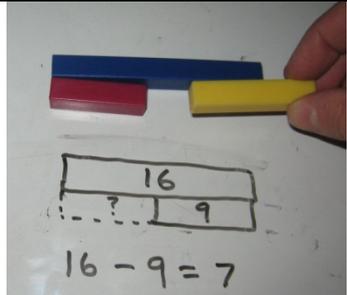
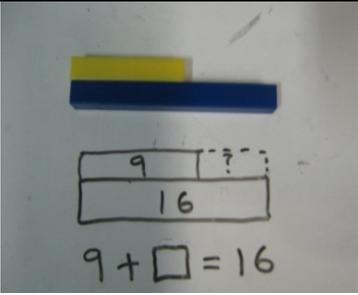
1. Subtraction of a two digit number, no exchange required.
2. Subtraction of a two digit number, exchange required.
3. Subtraction of a 3 digit number, one lot of exchange required.
4. Subtraction of a three digit number, 2 lots of exchange required.
5. Subtraction involving decimal numbers with up to 3 decimal places in a variety of contexts including money.

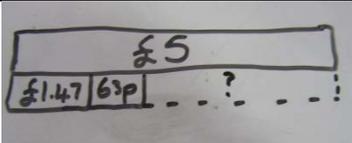
**Key Vocabulary:**

Take, take away, less, fewer, how many left, equals, partition, difference, count back, count on, less than, subtract, minus, exchange, decrease, tens, ones, inverse, tenths, hundredths, thousandths, decimal point, decimal.

### Addition and Subtraction as the Inverse

It is important that addition and subtraction are taught alongside one another so that children understand the relationship between them. This will help children to understand subtraction as “finding the difference”. Here are some pictorial ways of helping children to understand that addition and subtraction are the inverse of one another:

<p><b>Use of Numicon</b></p>			<p>Children can investigate how many different number sentences they can create using certain numicon apparatus. This can then be applied to missing number questions e.g. <math>4 + [ \quad ] = 10</math></p>
<p><b>Use of Cuisenaire and bar models</b></p>			 <p>It is essential that children do not associate particular Cuisenaire rods with numbers – they can represent <i>any</i> number. Children can investigate the inverse through Cuisenaire and bar models. The unknown quantity is always drawn with a dotted line. This method can later be applied to word problems and algebraic problems.</p>
<p><b>Use of bar models to solve word problems</b></p>	<p>Problem: Sarah buys a cheese sandwich for £1.47 and an apple pie for 63p. She pays with a £5 note. How</p>	<p>Children represent the problem pictorially using a bar model. It is then clear that they need to add £1.47 and 63p and then work out the difference between that answer and £5 (through use of counting on strategies).</p>	

	much change does she get?		
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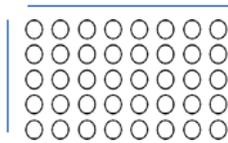
### Progression in Multiplication

These notes show the stages in building up to a compact, efficient method for multiplication. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage. Multiplication should be taught alongside its inverse, division.

<p><b>Stage 1</b></p> <p><b>Repeated Addition: Practical Multiplication</b></p> <p>Children need plenty of experience of multiplying using repeated addition with concrete objects and pictorial representations. Give children plenty of opportunities to count in equal groups. Give children plenty of problem solving activities involving counting equal sets or groups.</p>	<p><b>Children need to be able to:</b></p> <ul style="list-style-type: none"> <li>Count in 2s, 5s and 10s.</li> <li>Recognise equal sets.</li> </ul> <p><b>Key Vocabulary:</b> Groups of, lots of, altogether, equals, count, repeated addition.</p>
<p>e.g. How many legs on 5 teddies?</p>  <p><math>2 + 2 + 2 + 2 + 2 = 10</math></p>	<p>There are 3 sweets in a bag. How many sweets in 3 bags?</p>  <p><math>3 + 3 + 3 = 15</math></p>
<p><b>Stage 2</b></p> <p><b>Repeated Addition: Arrays</b></p> <p>Children will recognise multiplication as repeated addition and picture this as arrays.</p>	<p><b>Children need to be able to:</b></p> <ul style="list-style-type: none"> <li>Count in steps</li> <li>Understand multiplication as repeated addition</li> </ul>

e.g. Children can represent  $8 \times 5$  pictorially as (this can also be done using the 5 and 8 Cuisenaire rods):

8

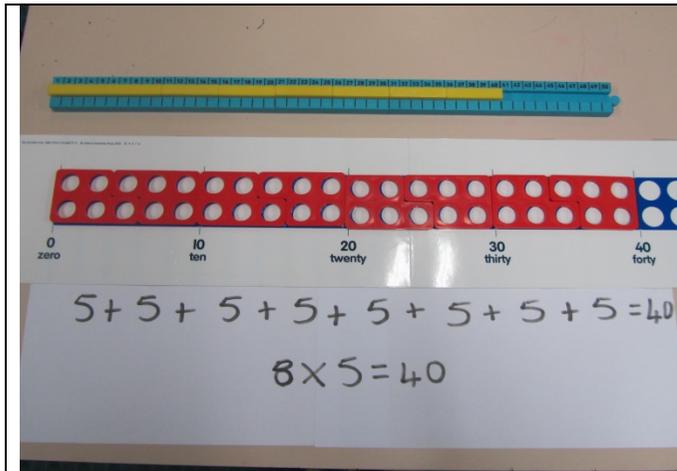


5

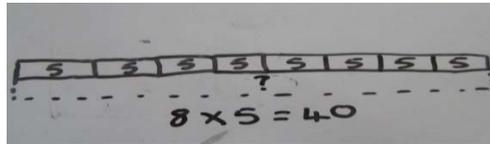
Children can then work this out using repeated addition.

### Repeated Addition : Number lines

To work out  $8 \times 5$  children can use numicon or Cuisenaire to create a number line:

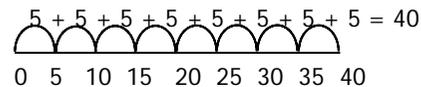


Children can record this as a bar model:



The dotted line shows the unknown quantity. Children could then replace the question mark with the number 40.

Children can then record this onto an empty number line:



$$8 \times 5 = 40$$

Children will then move on to using a number line to multiply larger numbers by using the distributive law (which allows numbers to be partitioned and each part to be multiplied separately; the parts then added to attain an answer).

- Understand that multiplication is commutative (multiplication can be done in any order).
- Solve one step multiplication problems.
- Understand multiplication as the inverse of division.

### Key Vocabulary:

Groups of, lots of, altogether, equals, count, repeated addition, sets of, row, column, multiply, times, \_\_\_ times as big as, array.

### Children need to be able to:

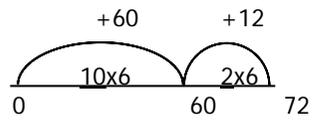
- Count in steps accurately.
- Understand multiplication as repeated addition.
- Be familiar with 2, 5, 10, 3, 4 multiplication tables.
- Solve one step problems involving multiplication.
- Understand multiplication to be the inverse of division.

### Key Vocabulary:

Groups of, lots of, altogether, equals, count, repeated addition, sets of, row, column, multiply, times, \_\_\_ times as big as, array, bar model, number line.

e.g.

$12 \times 6$



Children should become completely confident with this method before progressing onto the next stage.

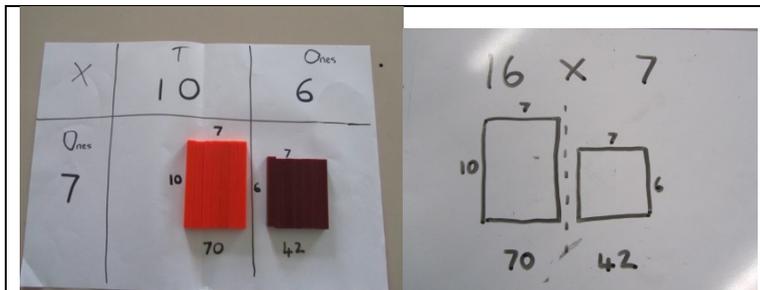
### Stage 3

#### Grid Method

The grid method should be introduced alongside children physically making an array to represent the calculation.

#### Two digit teens number multiplied by a single digit:

e.g.  $16 \times 7$



x	10	6	
7	70	42	112

$16 \times 7 = 112$

#### Two digit numbers by a single digit:

e.g.

$56 \times 7 = 392$

x	50	6	
7	350	42	392

Ensure that children understand the relationship between  $7 \times 5$  and  $50 \times 5$  and that they are not simply “adding a zero”.

#### Multiplying a 2 digit by a 2 digit number:

Children should partition both numbers and multiply each part. Children can then add the parts together, using column addition if needed:

e.g.  $56 \times 27 = 1512$

x	20	7	
50	1000	350	1350
6	120	42	162

#### Children need to be able to:

- Partition numbers
- Recall multiplication facts up to  $12 \times 12$
- Have a secure understanding of related multiplication facts e.g. from  $5 \times 7$  and place value knowledge, know  $50 \times 7$ ,  $50 \times 70$ ,  $700 \times 5$ .
- Have a secure understanding of place value.
- Add combinations of numbers mentally or using column addition.
- Solve two step problems involving multiplication.
- Understand multiplication to be the inverse of division.

#### Key Vocabulary:

Groups of, lots of, altogether, equals, count, repeated addition, sets of, row, column, multiply, times, \_\_\_ times as big as, array, bar model, number line, ten times bigger, 100 times bigger, multiple, product, inverse.

	1512
--	------

**Multiplying by a decimal:**

Children can also multiply by decimals using grid method.

x	30	6	0.8	
7	210	42	5.6	257.6

e.g.  $36.8 \times 7 = 257.6$

Children can then use column addition to add up the numbers.

**Children need to be able to:**

- Have a secure understanding of decimal place value.
- Understand the relationship between facts such as  $7 \times 8$  and  $0.7 \times 8$ .
- Have a secure knowledge of times tables facts up to  $12 \times 12$ .
- Use column addition to add decimal numbers.
- Solve two step problems involving multiplication.
- Understand multiplication to be the inverse of division.

**Key Vocabulary:**

Groups of, lots of, altogether, equals, count, repeated addition, sets of, row, column, multiply, times, \_\_\_ times as big as, array, bar model, number line, ten times bigger, 100 times bigger, 10 times smaller, 100 times smaller, multiple, product, inverse.

**Stage 4**

**Short Multiplication**

Once children have a *secure* understanding of the grid method, they can move on to short multiplication.

e.g.  $56 \times 7$

x	50	6	
7	350	42	392



$$\begin{array}{r} 56 \\ \times 7 \\ \hline 392 \end{array}$$

Introduce short multiplication alongside the grid method. Ask children to compare similarities and differences between the two methods. Unpick the steps to show how they are reduced from grid method.

**Children need to be able to:**

- Have secure knowledge of times tables up to  $12 \times 12$ .
- Have secure knowledge of place value, including decimal place value.
- Be able to multiply and divide decimals to 2dp by 10, 100 and 1000.
- Have a secure understanding of the grid method.
- Solve complex multistep problems involving multiplication.
- Understand multiplication to be the inverse of division.

**Long Multiplication**

Children can multiply two digit numbers by two or three digit numbers as follows:

e.g.  $56 \times 27 = 1512$

x	50	6
---	----	---



$$\begin{array}{r} 56 \\ \times 27 \\ \hline \end{array}$$

Introduce long multiplication alongside grid method. Unpick the

**Key Vocabulary:**

Groups of, lots of, altogether, equals, count, repeated addition, sets of, row, column, multiply, times, \_\_\_ times as big as, array, bar model, number line, ten times bigger,

20	1000	120	
7	350	42	1512

$$\begin{array}{r}
 \underline{\quad} \text{X } 27 \\
 3942 \quad (56 \times 7) \\
 1120 \quad (56 \times 20) \\
 \hline
 1512 \\
 \quad \quad 1
 \end{array}$$

steps to show how they are reduced from grid method.

Eventually children will be able to do this without writing out what they are multiplying at the side.

100 times bigger, 10 times smaller, 100 times smaller, multiple, product, inverse.

### Multiplying with decimals

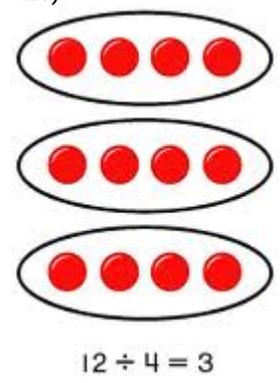
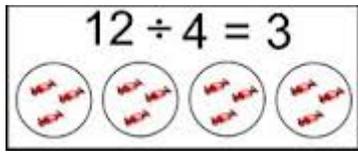
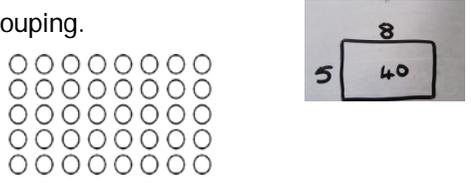
As children progress, they will be able to use the short multiplication method to multiply decimal numbers.

e.g.  $3.19 \times 8 = 12.52$

$$\begin{array}{r}
 \quad \quad 3 \quad . \quad 1 \quad 9 \\
 \text{X} \quad 8 \\
 \hline
 2 \quad 5 \quad . \quad 5 \quad 2 \\
 \hline
 \quad \quad 1 \quad 7
 \end{array}$$

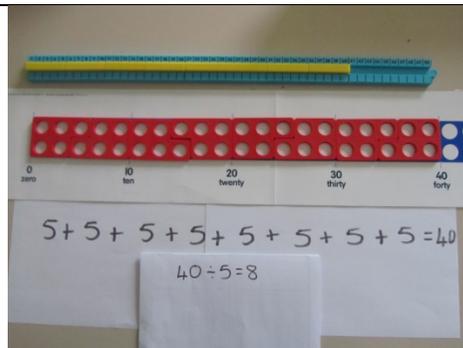
## Progression in Division

These notes show the stages in building up to a compact, efficient method for division. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage. Division should be taught alongside its inverse, multiplication.

<p><b>Stage 1</b>  <b>Grouping and Sharing: Practical Division</b>                  Children should have plenty of opportunity to use objects, diagrams and pictorial representations to solve problems involving both grouping <i>and</i> sharing.                  Children should be taught to understand the difference between grouping and sharing (how many groups of 2 can you make with 6 sweets? Share these 6 sweets between 2 people).                  Children should be able to find half of a group of objects by sharing it into 2 equal groups.</p>	<p>Children need to be able to:</p> <ul style="list-style-type: none"> <li>• Count in multiples of 2s, 5s and 10s.</li> </ul> <p><b>Key Vocabulary:</b>                  Share, share equally, one each, two each, group, groups of, lots of, half.</p>
<p>e.g. Group these 12 sweets into 4s (how many groups of 4 in 12?):</p> 	<p>e.g. Share these 12 sweets equally between 4 people:</p> 
<p><b>Stage 2</b>  <b>Repeated Addition: Arrays</b>                  Children should be introduced to using arrays for division at the same time as using them for multiplication.</p> <p>e.g. <math>40 \div 5</math> can be asked as how many 5s in 40? This can be linked back to grouping.                  Children can then draw this as an array:</p> 	<p>Children need to be able to:</p> <ul style="list-style-type: none"> <li>• Count in steps.</li> <li>• Understand division as grouping.</li> <li>• Understand that division is the inverse of multiplication.</li> <li>• Solve one step problems involving division.</li> </ul> <p><b>Key Vocabulary:</b>                  Share, share equally, one each, two each, group, groups of, lots of, half, array, divide, division.</p> <p>Children need to be able to:</p>
<p><b>Repeated Addition: Number lines</b>                  Children should be taught to link their array to a number line using practical apparatus.</p>	

Example without a remainder

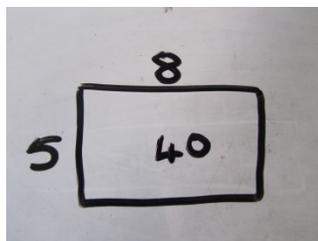
$40 \div 5 = 8$  (Phrase this as "how many 5s in 40?")



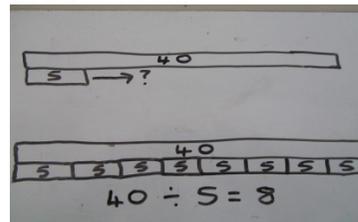
Children can use Cuisenaire or Numicon to work this out using grouping.



Children can then take the Cuisenaire from the rod track and rearrange it into an array.



Children should be taught to represent this as a bar model:



This can then be recorded on an empty number line:

$$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$$

0 5 10 15 20 25 30 35 40

$$40 \div 5 = 8$$

- Count in steps.
- Understand division as grouping.
- Understand that division is the inverse of multiplication.
- Be more familiar with times tables up to  $12 \times 12$ .
- Understand remainders.
- Derive larger multiples using known facts e.g.  $10 \times 3 = 30$  so  $20 \times 3 = 60$ .
- Add multiples mentally and work out differences.
- Solve one and two step problems involving division.

**Key Vocabulary:**

Share, share equally, one each, two each, group, groups of, lots of, half, array, divide, division, fraction, inverse, remainder.

Example with a remainder

This should first be done practically using Cuisenaire and Numicon as above. Children should also be able to represent it as a bar model as above.

$$38 \div 6 = 6 \text{ r } 2$$

$$6 + 6 + 6 + 6 + 6 + 6 + 2 = 6 \text{ sixes with a remainder}$$

0 6 12 18 24 30 36 38

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

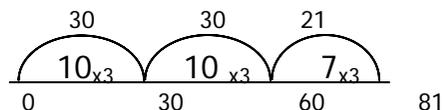
**Fractions of quantities (where the numerator is 1 and the denominator is under 12) should be introduced alongside division. (For example, children can find one fifth of 40 and realize that this is the same as  $40 \div 5$ ).**

Examples for a larger numbers

For larger numbers it would be inefficient to count in single multiples so bigger jumps need to be recorded using known facts.

e.g. without a remainder

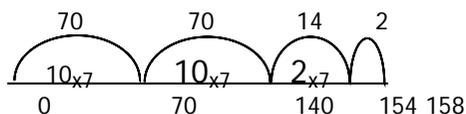
$$81 \div 3 = 27$$



This is done by working out the numbers of threes in each jump as you go along (10 threes are 30, another 10 threes makes 60, and another 7 threes makes 81. That's 27 threes altogether).

e.g. with a remainder

$$158 \div 7$$



10 sevens are 70, add another 10 sevens is 140, add 2 more sevens is 154 add 2 makes 158. So there are 22 sevens with a remainder of 2.

The remainder is indicated above the jump rather than inside it, so that children do not mistakenly add 10, 10, 2 and 2 and get an answer of 24.

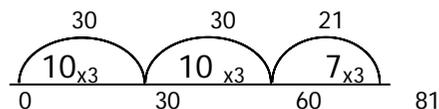
### Stage 3

#### Short Division

When children have a secure understanding of all the previous steps they can move onto short division.

#### No remainder

$$81 \div 3 = 27$$



$$81 \div 3$$

$$\begin{array}{r} 27 \\ 3 \overline{) 81} \end{array}$$

Children use their knowledge of the 3 times table to find, "How many 3s in 80 where the answer is a multiple of 10?" This gives 20 threes (since 30 threes would be too many), with 20 remaining (2 tens are carried over to the next column) Now ask: 'How many threes in 21'.

Show children the two methods side by side. Ask children to unpick the steps of the short division method and note the similarities and differences between that and the number line method.

**Remainders:** Once secure, children can use this to find remainders, then remainders as fractions and then remainders as decimals:

#### Children need to be able to:

- Recall all multiplication facts and related division facts up to  $12 \times 12$ .
- Understand place value and use this to divide and multiply by 10, 100 and 100.
- Relate division to fractions.
- Understand division and multiplication as the inverse.
- Find fractions of quantities where the numerator is 1.

#### Key Vocabulary:

Share, share equally, one each, two each, group, groups of, lots of, half, array, divide, division, fraction, inverse, remainder, quotient (the answer), divisor (number you are dividing by), dividend (number you are dividing into), decimal.

$$284 \div 6 = 284 \text{ r}2$$

$$\begin{array}{r} 47 \text{ r}2 \\ 6 \overline{) 284} \end{array}$$

$$284 \div 6 = 284 \frac{2}{6}$$

$$\begin{array}{r} 47 \text{ r}2 \\ 6 \overline{) 284} \end{array}$$

$$284 \div 4 = 70.5$$

$$\begin{array}{r} 70.5 \\ 4 \overline{) 284.200} \end{array}$$

#### Stage 4

#### Long Division

When dividing by numbers larger than 12, children will need to use long division.

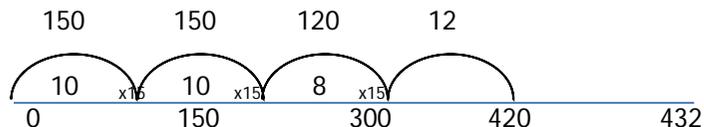
#### Chunking on a number line

e.g.  $432 \div 15 = 28 \text{ r}12$

First children should write out the 15 times table up to ten lots of 15:

1	2	3	4	5	6	7	8	9	10
15	30	45	60	75	90	105	120	135	150

Encourage children to do this using known facts. Start with one 15 and double it to get two 15s. Double this to get four 15s and double again to get eight 15s. Then find ten 15s and halve to get five 15s. Children can then fill in the rest.



#### Formal Long Division

Move on to this *only* if children are secure with the number line method above. Introduce it along with the example on the number line so that children can see the similarities and differences and discuss these.

#### Example with a remainder

$$\begin{array}{r} 28 \text{ r}12 \\ 15 \overline{) 432} \\ \underline{30} \phantom{0} \\ 132 \end{array}$$

#### Example with the remainder as a fraction

$$\begin{array}{r} 28 \text{ r}12 \\ 15 \overline{) 432} \\ \underline{30} \phantom{0} \\ 132 \end{array}$$

#### Example with the remainder as a decimal

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \phantom{00} \\ 132 \phantom{0} \end{array}$$

#### Children need to be able to:

- Recall all multiplication facts and related division facts up to 12 x 12.
- Understand place value and use this to divide and multiply by 10, 100 and 100.
- Relate division to fractions.
- Understand division and multiplication as the inverse.
- Understand fractions
- Understand decimals and decimal place value.
- Find fractions of quantities where the numerator and denominator could be any number.

#### Key Vocabulary:

Share, share equally, one each, two each, group, groups of, lots of, half, array, divide, division, fraction, inverse, remainder, quotient (the answer), divisor (number you are dividing by), dividend (number you are dividing into), decimal.

$\begin{array}{r} 120 \\ 12 \end{array}$ <p><math>432 \div 15 = 28 \text{ r}12</math></p>	$\begin{array}{r} 120 \\ 12 \end{array}$ <p><math>432 \div 15 = 28 \frac{12}{15}</math> Children can simplify <math>\frac{12}{15}</math> to <math>\frac{4}{5}</math></p>	$\begin{array}{r} 120 \\ 120 \\ 120 \\ \hline 0 \end{array}$ <p><math>432 \div 15 = 28.8</math></p>	
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